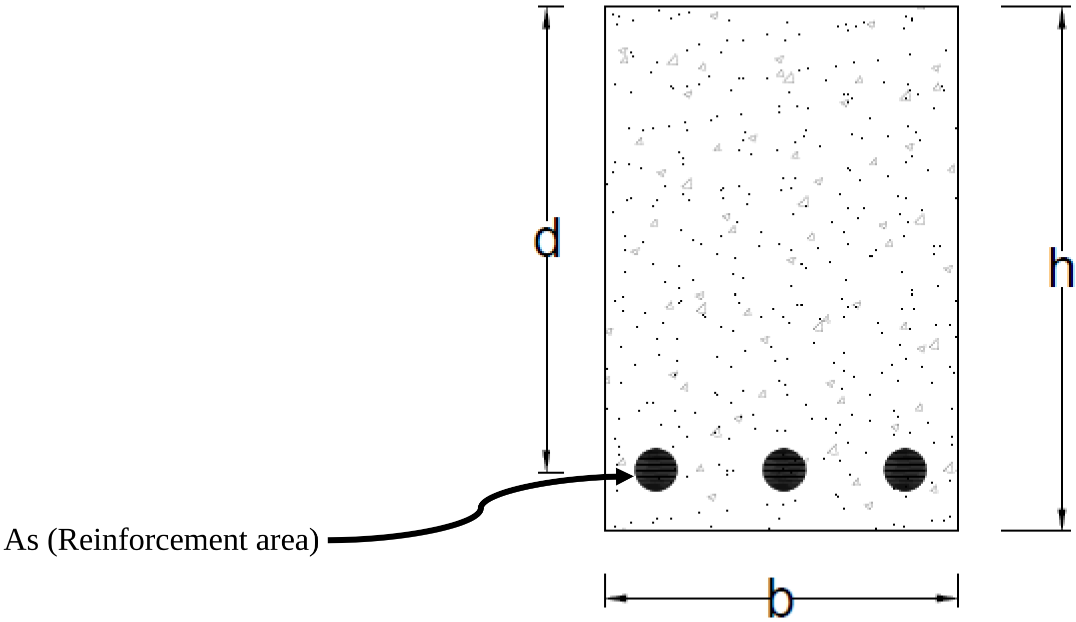
Analysis of rectangular beams with tension reinforcement

- Generally , in the analysis problem the following information are known:

1. Beam dimensions and reinforcement (b,h,d and As).

2. Materials strength (fy and Fc`).



And following information are required

- Check the adequacy of the section according to ACI requirement.
- Compute design moment ØMn.
- Compute the maximum live or dead load.

Procedure Analysis for Rectangular Beams with tension Reinforcement (Singly reinforcement)

1. Calculate $\rho = \frac{As}{bd}$

   Where As = n $\times \frac{\pi}{4} \times D^2$

   N = number of bars

   D = diameter of reinforcement bar

   Check if the provided ρ is in agreement with ACI requirements.

   $\rho \le \rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u + 0.004}$

   $\epsilon_u = 0.003$

   $As \ge As_{\text{minimum}} = \frac{0.25\sqrt{fc`}}{fy} b_w d \ge \frac{1.4}{fy} b_w d$

2. Calculate Ø

   $a = \frac{Asfy}{0.85fc`b}$

   $c = \frac{a}{\beta 1}$

   - according to ACI code β1 can be calculated from table below

Table 22.2.2.4.3—Values of β_1 for equivalent rectangular concrete stress distribution

f_c', MPa	β_1	
$17 \le f_c' \le 28$	0.85	(a)
$28 < f_c' < 55$	$0.85 - \frac{0.05(f_c' - 28)}{7}$	(b)
$f_c' \ge 55$	0.65	(c)

$\epsilon t = \frac{dt - c}{c} \epsilon u$ where: $\epsilon u = 0.003$

- If $\epsilon t \ge 0.005$, then Ø=0.9
- If $\epsilon t < 0.005$ then

Ø=0.483+83.3×∈t

3. Calculate ∅Mn

   ∅Mn can be calculated from:

   $$\emptyset Mn = \emptyset A_s f_y \left(d - \frac{a}{2}\right)$$

   Or

   $$\emptyset Mn = \emptyset 0.85 f_c' ab \left(d - \frac{a}{2}\right)$$

   Or

   $$\emptyset Mn = \emptyset \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

4. Find Mu and compare it with φMn

   If φMn≥Mu the section is Ok

   If φMn<Mu the section is not Ok

Concrete Cover

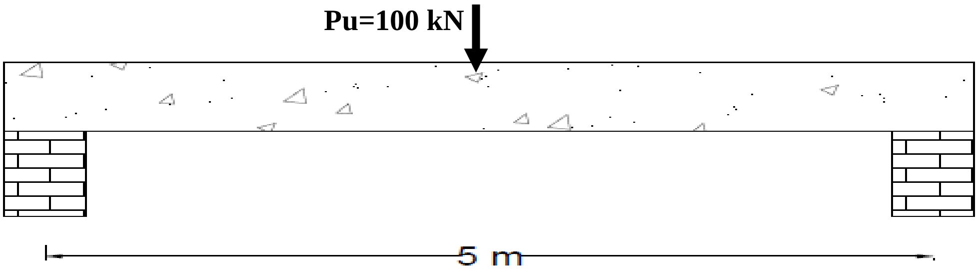
To provide the steel with adequate concrete protection against corrosion, the designer must maintain a certain minimum thickness of concrete to cover outside of the outermost steel.

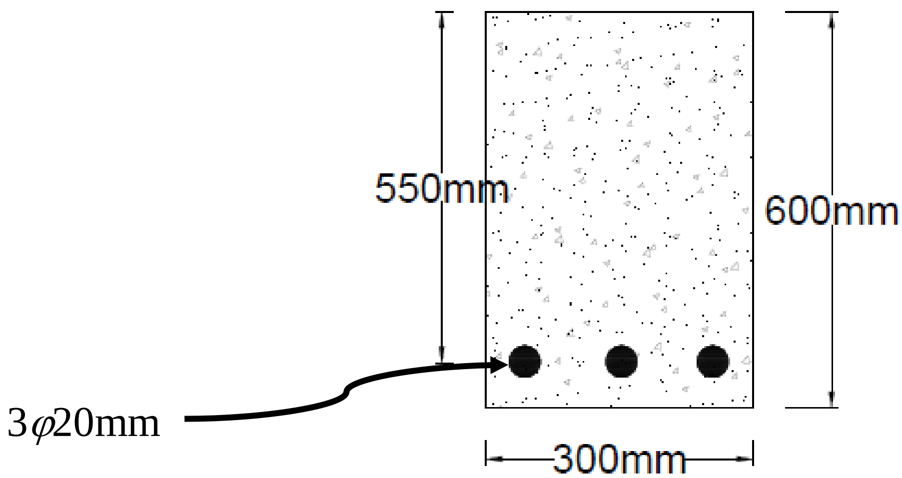
Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

Concrete exposure	Member	Reinforcement	Specified cover, mm
Cast against and permanently in contact with ground	All	All	75
Exposed to weather or in contact with ground	All	No. 19 through No. 57 bars	50
		No. 16 bar, MW200 or MD200 wire, and smaller	40
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 43 and No. 57 bars	40
		No. 36 bar and smaller	20
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	40

- As a general case, requirement for beams (that not exposed to weather)=**40 mm**

Example 1: Check the adequacy of the beam shown below according to ACI requirement, use fc`=25 Mpa, fy=400 Mpa, neglect the self-weight





Solution:

$As = n \times \frac{\pi}{4} \times D^2 = 3\times \frac{\pi}{4} \times 20^2 = 942\ mm^2$

$\rho = \frac{As}{bd} = \frac{942}{300*550} = 5.71\times 10^{-3}$

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u+0.004} = 0.85\times0.85\times \frac{25}{400} \times \frac{0.003}{0.003+0.004} = 19.4\times10^{-3}$

$\rho < \rho_{max}$ O.k

$As_{minimum} = \frac{1.4}{fy} b_w \times d = \frac{1.4}{400} \times300\times550 = 525\ mm^2$

$As > As_{minimum}$ o.k

$A = \frac{As*fy}{0.85fc`*b} = \frac{942\times400}{0.85\times25\times300} = 59.1\ mm$

$c = \frac{a}{\beta 1} = \frac{59.1}{0.85} = 69.5\ mm$

$\epsilon t = \frac{dt-c}{c} \epsilon u = \frac{550-69.5}{69.5} \times 0.003 = 20.7\times10^{-3}$

$\epsilon t > 0.005$

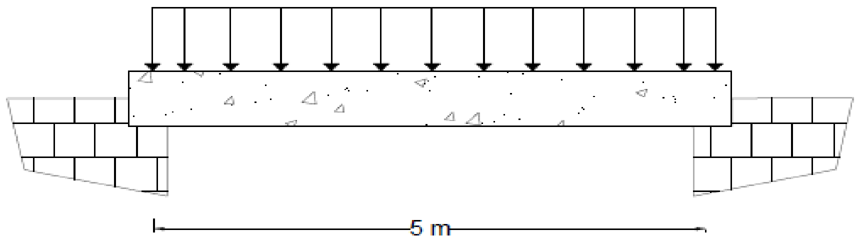
$\emptyset = 0.9$

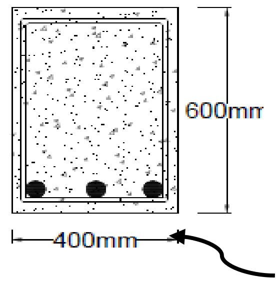
$\emptyset Mn = \emptyset As \times fy(d - \frac{a}{2}) = 0.9 \times 942 \times 400 \times (550 - \frac{59.1}{2}) \times 10^{-6} = 176\ kN.m$

$Mu = \frac{Pu\times L}{4} = \frac{100\times5}{4} = 125\ kN.m$ $\emptyset Mn > Mu$ the section is O.k ■

Example 2: Check the adequacy for the beam below according to ACI requirement, if the beam is subjected to uniform dead load (3) kN/m **(without self-weight)** and uniform live load (2) kN/m, use fc`=28 Mpa and fy=420 Mpa

W_D=3 kN/m (without self-weight), W_L=2 kN/m





Solution:

As = n × $\frac{\pi}{4}$ × D^2=3× $\frac{\pi}{4}$ × 25^2=1472.6 mm^2

d=h− cover−stirrups− $\frac{\varphi bar}{2}$ = 600−40−10− $\frac{25}{2}$ = 537.5 mm

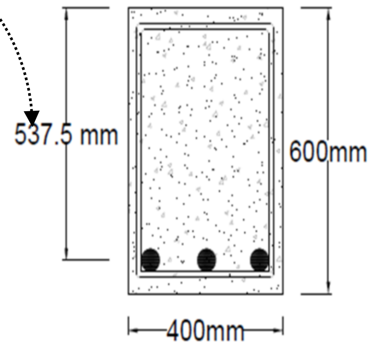
$\rho = \frac{As}{bd} = \frac{1472.6}{400\times537.5} = 6.85\times10^{-3}$

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u+0.004} = 0.85\times 0.85\times \frac{28}{420} \times \frac{0.003}{0.003+0.004} = 0.0206$

$\rho < \rho_{max}$ O.k

As $_{minimum}$=$\frac{1.4}{fy}$$b_w$×d = $\frac{1.4}{420}$ ×400×537.5 = 716.67 mm^2

As > As $_{minimum}$



a= $\frac{Asfy}{0.85fc`b} = \frac{1472.6\times420}{0.85\times28\times400}$ = 64.96 mm

c= $\frac{a}{\beta1} = \frac{64.96}{0.85}$ =76.43 mm

$\epsilon t = \frac{dt-c}{c}\epsilon u = \frac{537.5-76.43}{76.43} \times 0.003 = 0.018$

ϵt = 0.018> 0.005

Ø=0.9

ØMn= ØAsfy(d−$\frac{a}{2}$) = 0.9×1472.6 ×420× (537.5 − $\frac{64.96}{2}$) ×10^{-6} = 281.1 kN.m

Find Mu

Mu=$\frac{Wu\times\ell^2}{8}$

Wu=1.2W_D+1.6W_L

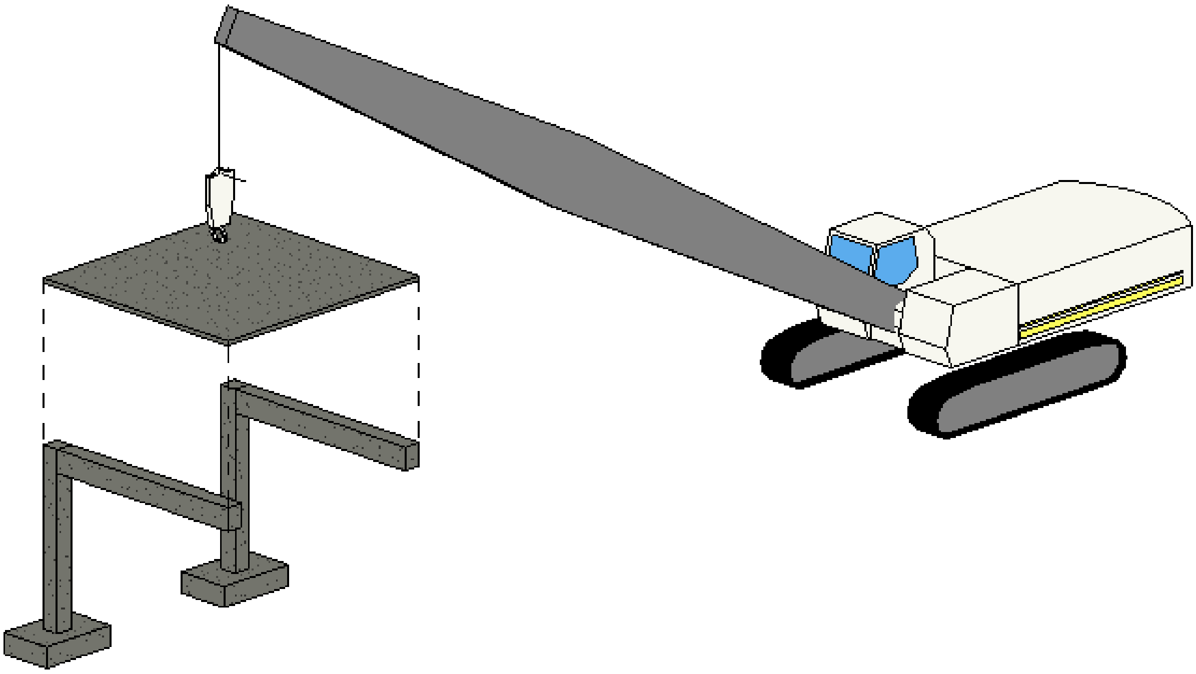
$W_{D\ self\text{-}weight}$ = γ × b × d=24 × 0.4 × 0.6 =5.76 kN/m

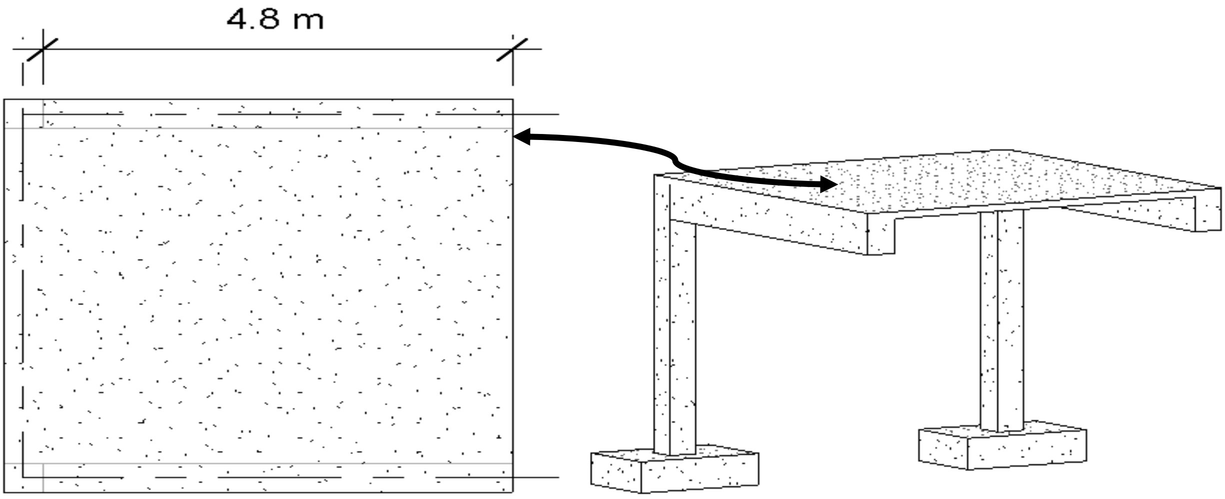
WD total=5.76 + 3=8.76 kN/m

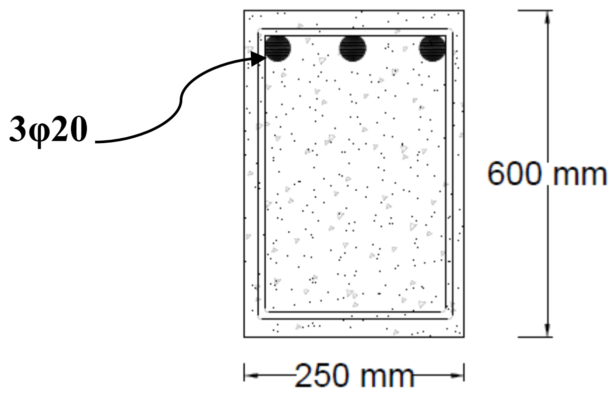
Wu=1.2 × 8.76 +1.6 × 2=13.712 kN/m

Mu=$\frac{Wu\times\ell^2}{8} = \frac{13.712\times5^2}{8}$ = 42.85 kN.m ØMn> Mu section is O.K. ■

Example: check the adequacy of the cantilever shown below; the cantilever is subjected to uniform dead load (3) kN/m **(include self-weight)** and uniform live load (4) kN/m, fc`=28 Mpa and fy=420 Mpa







Solution:

$As = n\times \frac{\pi}{4} \times D^2 = 3\times \frac{\pi}{4} \times 20^2 = 942\ mm^2$

$d = 600 - 40 - 10 - \frac{20}{2} = 540\ mm$

$\rho = \frac{As}{bd} = \frac{942}{250\times540} = 6.98\times10^{-3}$

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u + 0.004} = 0.85\times0.85\times \frac{28}{420} \times \frac{0.003}{0.003+0.004} = 0.0206$

$\rho < \rho_{max}$ O.k

$As_{minimum} = \frac{1.4}{fy} b_w \times d = \frac{1.4}{420} \times 250 \times 540 = 450\ mm^2$

$As > As_{minimum}$ o.k

$a = \frac{As\times fy}{0.85fc`\times b} = \frac{942\times420}{0.85\times28\times250} = 66.5\ mm$

$c = \frac{a}{\beta1} = \frac{66.5}{0.85} = 78.23\ mm$

$\epsilon t = \frac{dt-c}{c}\epsilon u = \frac{540-78.23}{78.23} \times 0.003 = 0.0177$

$\epsilon t > 0.005$

$\therefore \emptyset = 0.9$

$\emptyset Mn = \emptyset As f_y (d - \frac{a}{2}) = 0.9 \times 942 \times 420 \times \left(540 - \frac{66.5}{2}\right) \times 10^{-6} = 180.44\ kN.m$

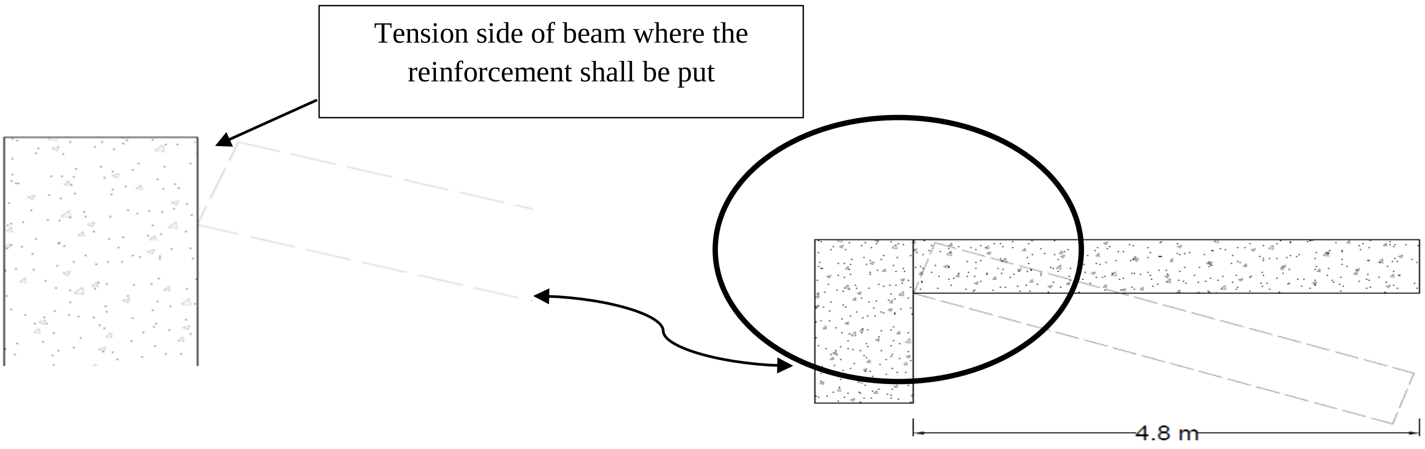
Calculate Mu

$Mu = \frac{Wu\times\ell^2}{2}$

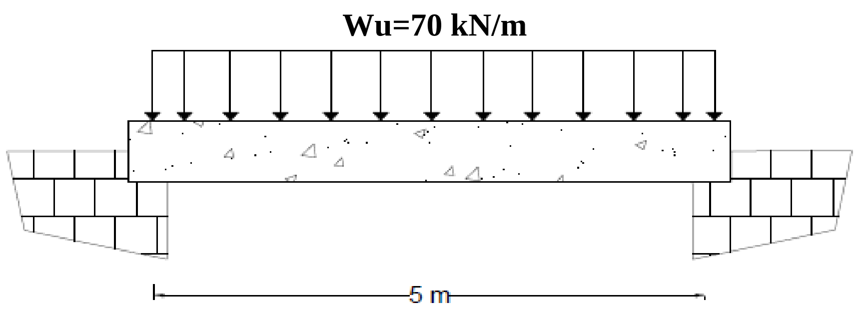
$Wu = 1.2W_D + 1.6W_L = 1.2\times3 + 1.6\times4 = 10\ kN/m$

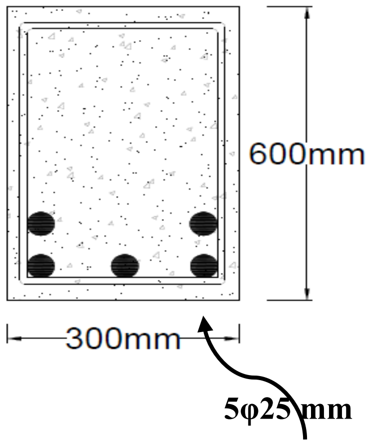
$Mu = \frac{Wu\times\ell^2}{2} = \frac{10\times4.8^2}{2} = 115.2\ kN.m$

$\emptyset Mn > Mu$ section is O.K. ■



Example: check the adequacy of a simply supported beam shown in figure below when subjected to a factored load of Wu=70 kN/m **(including self-weight),** use $fc`$=28 Mpa and fy=420 Mpa





Solution:

$As= n* \frac{\pi}{4} * D^2 = 5*\frac{\pi}{4} * 25^2 = 2454\ mm^2$

$d = 600-40-10-25-\frac{25}{2} = 512\ mm$

$\rho = \frac{As}{bd} = \frac{2454}{300*512} = 0.0159$

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u + 0.004} = 0.85*0.85*\frac{28}{420} * \frac{0.003}{0.003+0.004} = 0.0206$

$\rho < \rho_{max}$ O.k

$As_{minimum} = \frac{1.4}{fy} b_w * d = \frac{1.4}{420} * 300 * 512 = 512\ mm^2$

$As > As_{minimum}$ o.k

$a = \frac{As*fy}{0.85fc`*b} = \frac{2454*420}{0.85*28*300} = 144\ mm$

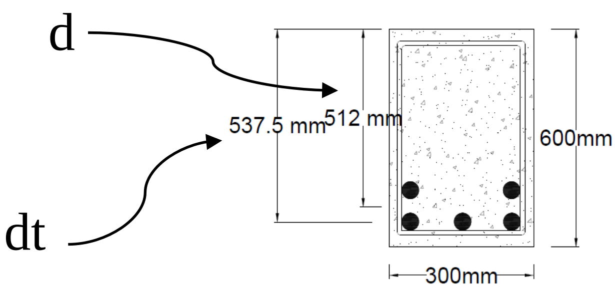
$c = \frac{a}{\beta 1} = \frac{144}{0.85} = 169.83\ mm$

$\epsilon t = \frac{dt-c}{c} \epsilon u = \frac{537.5-169.83}{169.83} * 0.003 = 6.5*10^{-3} > 0.005$

Then $\emptyset = 0.9$

$\emptyset Mn = \varphi As*fy(d-\frac{a}{2}) = 0.9*2454*420*(512-\frac{144}{2})*10^{-6} = 408\ kN.m$

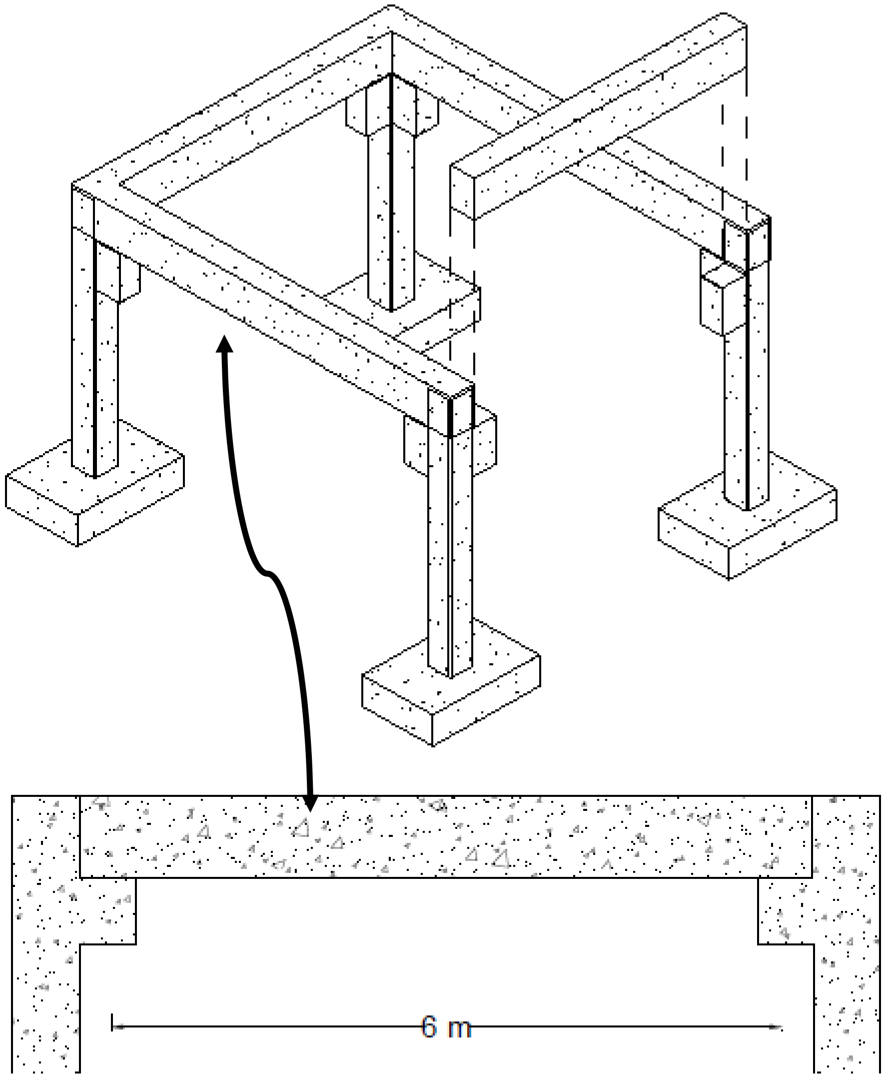
$Mu = \frac{Wu*\ell^2}{8} = \frac{70*5^2}{8} = 218.75\ kN.m$ $\varphi Mn > Mu$ section is O.K. ■



Example: for the precast beam shown in Figure below, the designer intended to use 4φ20 ,check the adequacy of the beam according to ACI requirement , the beam is subjected to uniform dead load (15) kN/m **(with self-weight)** and uniform live load (20) kN/m

Assume in your solution:

- The beam is simply supported
- Fc`=28 Mpa and fy=420 Mpa
- Single layer of reinforcements
- Beam with 250 mm and effective depth 500 mm



Solution:

As = n* $\frac{\pi}{4} * D^2$ = 4*$\frac{\pi}{4} * 20^2$=1256.63 mm^2

$\rho = \frac{As}{bd} = \frac{1256.63}{250*500} = 0.01$

$\rho_{max} = 0.85\beta_1 \frac{fc`}{fy} \frac{\epsilon u}{\epsilon u + 0.004} = 0.85*0.85*\frac{28}{420} * \frac{0.003}{0.003+0.004} = 0.0206$

$\rho < \rho_{max}$ O.k

$As_{minimum}=\frac{1.4}{fy}b_w*d=\frac{1.4}{420}*250*500=416.7\ mm^2$

$As > As_{minimum}$ o.k

$a=\frac{As*fy}{0.85fc`*b}=\frac{1256.63*420}{0.85*28*250}=88.7\ mm$

$c=\frac{a}{\beta1}=\frac{88.7}{0.85}=104.35\ mm$

$\epsilon t=\frac{dt-c}{c}\epsilon u=\frac{500-104.35}{104.35}*0.003=0.0113 > 0.005$

Then $\emptyset=0.9$

$\emptyset Mn=\emptyset As*fy(d-\frac{a}{2})=0.9*1256.63*420*(500-\frac{88.7}{2})*10^{-6}=216.43\ kN.m$

Calculate Mu

$W_U=1.2W_D+1.6W_L=1.2*15+1.6*20=50\ kN/m$

$Mu=\frac{Wu*\ell^2}{8}=\frac{50*6^2}{8}=225\ kN.m$

$\varphi Mn<Mu$ the section **is not O.K** ■

- beam dimensions must be increasing or using more reinforcement area .

Example: a rectangular beam with a width of 305 mm and an effective depth of 444 m. it is reinforced with 4φ29mm (assume A_{bar}=645 mm^2) if fc`=27.5 Mpa and fy=414 Mpa. Check the beam adequacy and compute its design strength according to ACI requirements.

Solution:

$As=n\times A_{bar}=4\times645=2580\ mm^2$

$\rho=\frac{As}{bd}=\frac{2580}{305*444}=0.019$

$\rho_{max}=0.85\beta_1\frac{fc`}{fy}\frac{\epsilon u}{\epsilon u+0.004}=0.85\times0.85\times\frac{27.5}{414}\times\frac{0.003}{0.003+0.004}=0.0205$

$\rho<\rho_{max}$ O.k

$As_{minimum}=\frac{1.4}{fy}b_w\times d=\frac{1.4}{414}\times305\times444=458\ mm^2$

$As > As_{minimum}$ o.k

$a=\frac{As\times fy}{0.85fc`\times b}=\frac{2580\times414}{0.85\times27.5\times305}=150\ mm$

$c=\frac{a}{\beta1}=\frac{150}{0.85}=176\ mm$

$\epsilon t=\frac{dt-c}{c}\epsilon u=\frac{444-176}{176}\times0.003=0.00457<0.005$

Then:

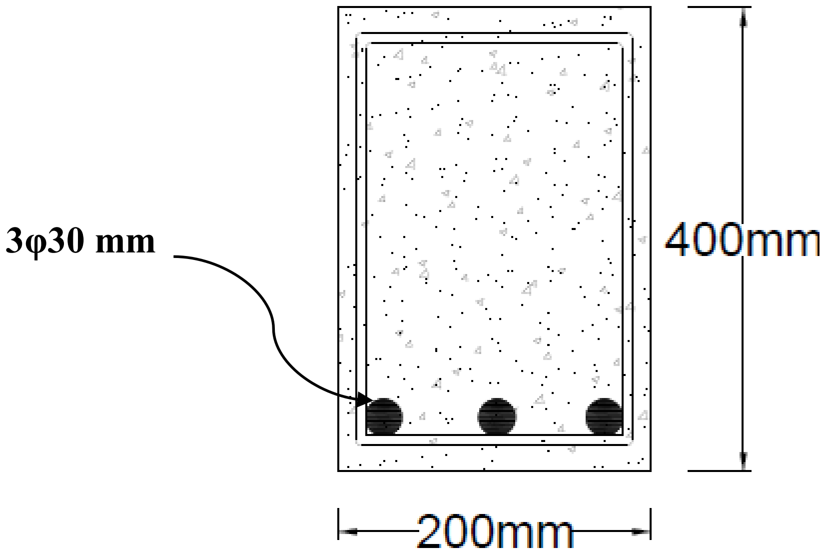
$\emptyset=0.483+83.3\times\epsilon t$

$\emptyset=0.483+83.3\times0.00457$

$\emptyset=0.86$

$\emptyset Mn=\emptyset As\times fy(d-\frac{a}{2})=0.86\times2580\times414\times(444-\frac{150}{2})\times10^{-6}=339\ kN.m$ ■

Example: Check the adequacy of the beam shown below according to ACI requirement, fc`=28 Mpa ,fy=420 Mpa



Solution:

As = n$\times \frac{\pi}{4} \times D^2$ = 3$\times \frac{\pi}{4} \times 30^2$ = 2120.57 mm^2

d = 400−40−10−15 = 335 mm

$\rho = \frac{As}{bd} = \frac{2120.57}{200*335} = 0.03165$

$\rho > \rho_{max}$

The section **is not O.K** and may not be used according to ACI requirements ■